Bellwork: 2/26/13
List all the factors of...

1) 48	2) 81	3) 72
1, 48	1, 81	1, 72
2, 24	3, 27	2, 36
3, 16	9, 9	3, 24
4, 12		4, 18
6, 8		6, 12
		8, 9

ALGEBRA 2 NAME ____________________
NOTES 5.5 DATE: ____________ PD______

THE RATIONAL ZERO TEST - FINDING ACTUAL ZEROS

The following test will allow you to determine ***which*** numbers to consider as possible roots.

> ***RATIONAL ZERO TEST:***
> If polynomial *f(x)* has **integer** coefficients, then every ***rational zero*** of this polynomial has the form:
>
> $\frac{p}{q}$ where $\begin{cases} p \text{ is a factor of the constant term of } f(x) \text{ and} \\ q \text{ is a factor of the leading coefficient of } f(x) \end{cases}$

Example 1: Find the ***possible rational roots (zeros)*** of $f(x) = 2x^3 + 3x^2 - 8x + 3$.

Constant term is 3 ➔ Factors: $\pm1, \pm3$ ➔ p

Leading coefficient ➔ Factors: $\pm1, \pm2$ ➔ q

Possible rational roots: $\frac{p}{q} = \frac{\pm1, \pm3}{\pm1, \pm2} = \boxed{\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}}$

There are 8 *possible rational* roots, however we know that this polynomial has *at most* 3 roots. This means that no more than 3 of these numbers will work.

Use synthetic division to test the possible rational roots to ***find the actual zeros***:

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & & 2 & 5 & -3 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

➔ $f(x) = (x-1)(2x^2 + 5x - 3)$

$= (x-1)(2x-1)(x+3)$

$-6x^2$

$(2x^2 + 6x)(-1x - 3)$

$2x(x+3)(-1(x+3)$

Actual Roots: $1, \frac{1}{2}, -3$

$\boxed{\text{Roots: } 1, \frac{1}{2}, -3}$

$$\begin{array}{r|rrrr} -3 & 3 & 4 & -13 & 6 \\ & & -9 & 15 & -6 \\ \hline & 3 & -5 & 2 & \boxed{0} \end{array}$$

Example 2: Find the actual zeros of $f(x)=3x^3+4x^2-13x+6$.

$\frac{1, 2, 3, 6}{1, 3}$

Maximum # of roots: 3

Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$

Factored form: $(x-1)(3x-2)(x+3)$

Actual roots: $1, \frac{2}{3}, -3$

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Work:

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -13 & 6 \\ & \downarrow & 3 & 7 & -6 \\ \hline & 3 & 7 & -6 & \boxed{0} \end{array}$$

$3x^2+7x-6$

$(3x^2+9x)(-2x-6)$

$3x(x+3)-2(x+3)$

$(3x-2)(x+3)$

$3x-2=0$

$3x=2$

$x=\frac{2}{3}$

$x+3=0$

$x=-3$

Note: With a 3rd degree polynomial, you only need to find 1 root using synthetic division. Then factor or use quadratic formula to find remaining roots of the polynomial.

Example 3: Find the actual zeros of $f(x)=-2x^4+13x^3-21x^2+2x+8$.

$\frac{1\ 2\ 4\ 8}{1\ 2}$

Maximum # of roots: 4

Possible rational roots: $\pm 1\ \pm 2\ \pm 4\ \pm 8\ \pm\frac{1}{2}$

Factored form: $-1(x-4)(x-1)(2x+1)(x-2)$

Actual roots: $4, 1, -\frac{1}{2}, 2$

Work:

$$\begin{array}{r|rrrrr} 4 & -2 & 13 & -21 & 2 & 8 \\ & +\downarrow & -8 & 20 & -4 & -8 \\ \hline 1 & -2 & 5 & -1 & -2 & \boxed{0} \\ & \downarrow & -2 & 3 & 2 & \\ \hline & -2 & 3 & 2 & \boxed{0} & \end{array}$$

$-2x^2+3x+2$ $\quad -4$

$-1(2x^2-3x-2)$

$(2x^2-4x)(+1x-2)$

$2x(x-2)+1(x-2)$

$-1(2x+1)(x-2)$

$2x+1=0$

$2x=-1$

$x-2=0$

$x=2$

Homework: pg 316 #9-17

just find the possible zeros